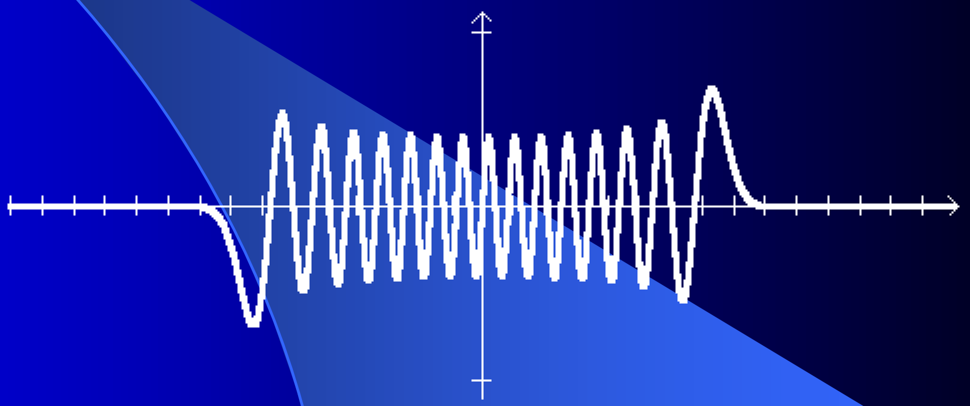
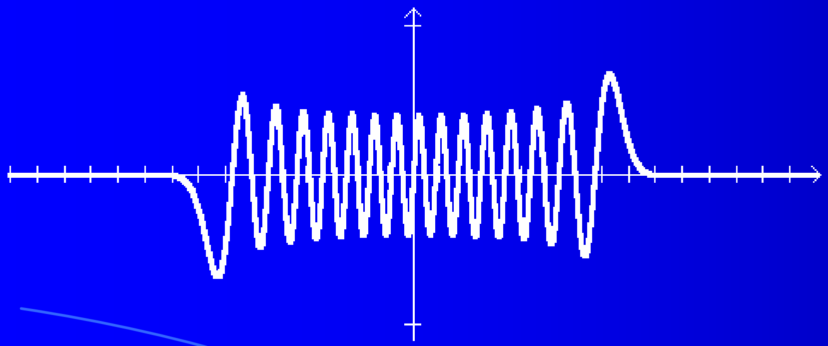
Лекция 7

Проекционный метод обращения преобразования Фурье с использованием функций Эрмита



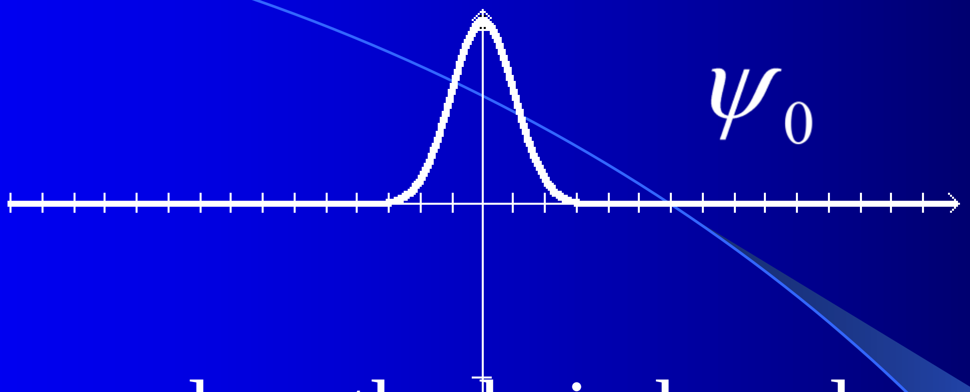


Проекционный метод обращения преобразования Фурье с использованием функций Эрмита

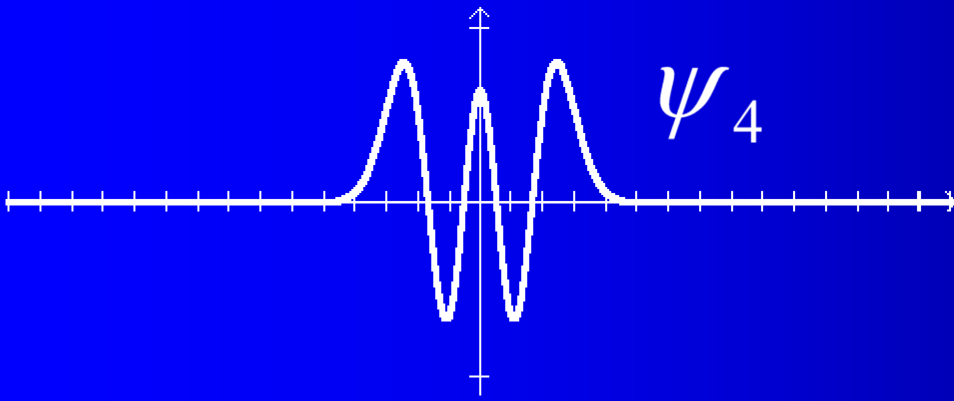
Outline:

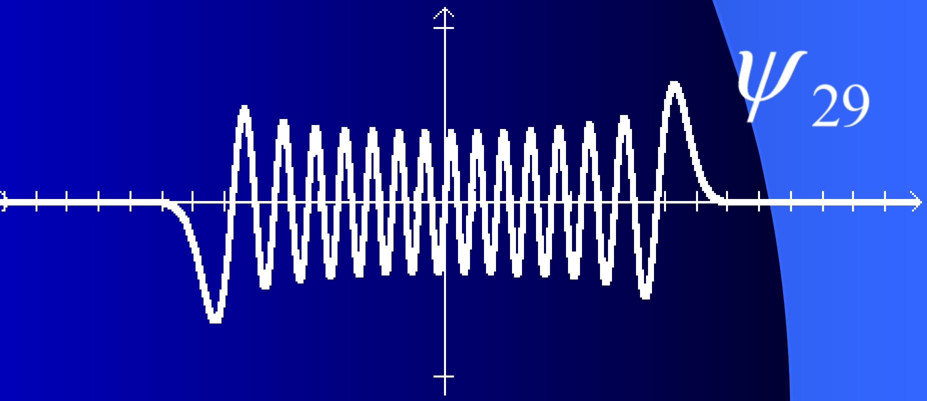
- Projection Method (Hermite series approach)
- Applications
  1. Image filtering and deblocking by projection filtering
  2. Image matching
  3. Texture matching
  4. Low-level methods for audio
  5. Hermite foveation

Hermite transform



The proposed methods is based on the features of Hermite functions. An expansion of signal information into a series of these functions enables one to perform information analysis of the signal and its Fourier transform at the same time.





A) $$\hat{\psi}_n = i^n \psi_n$$

B) They derivate a full orthonormal in $L_2(-\infty, \infty)$ system of functions.

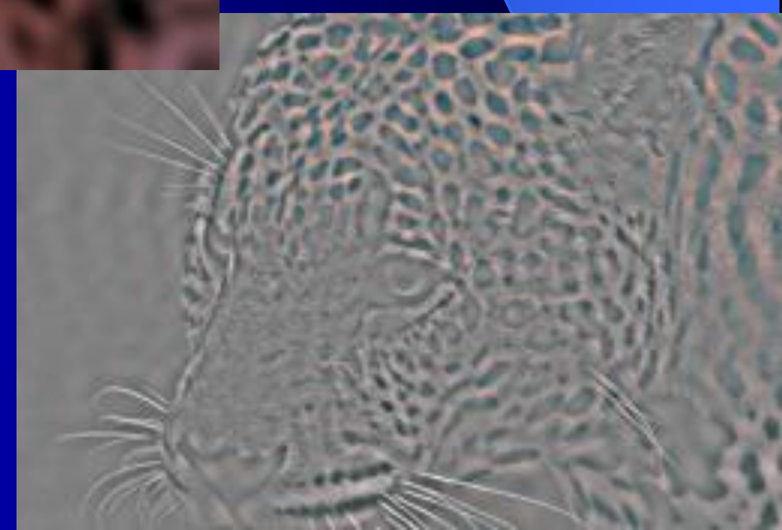
The Hermite functions are defined as:

$$\psi_n(x) = \frac{(-1)^n e^{x^2/2}}{\sqrt{2^n n! \sqrt{\pi}}} \cdot \frac{d^n (e^{-x^2})}{dx^n}$$

Original image

2D decoded image by 45 Hermite functions at the first pass and 30 Hermite functions at the second pass

Difference image (+50% intensity)



Original image

2D decoded image by 90 Hermite functions at the first pass and 60 Hermite functions at the second pass

Difference image (+50% intensity)

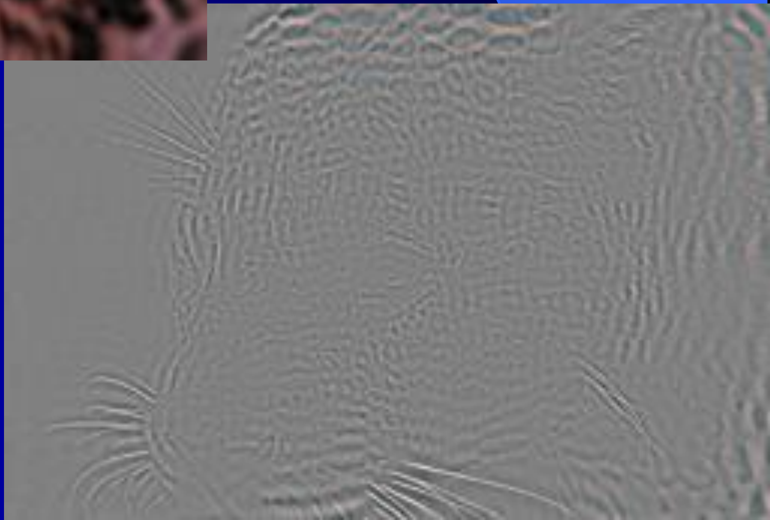
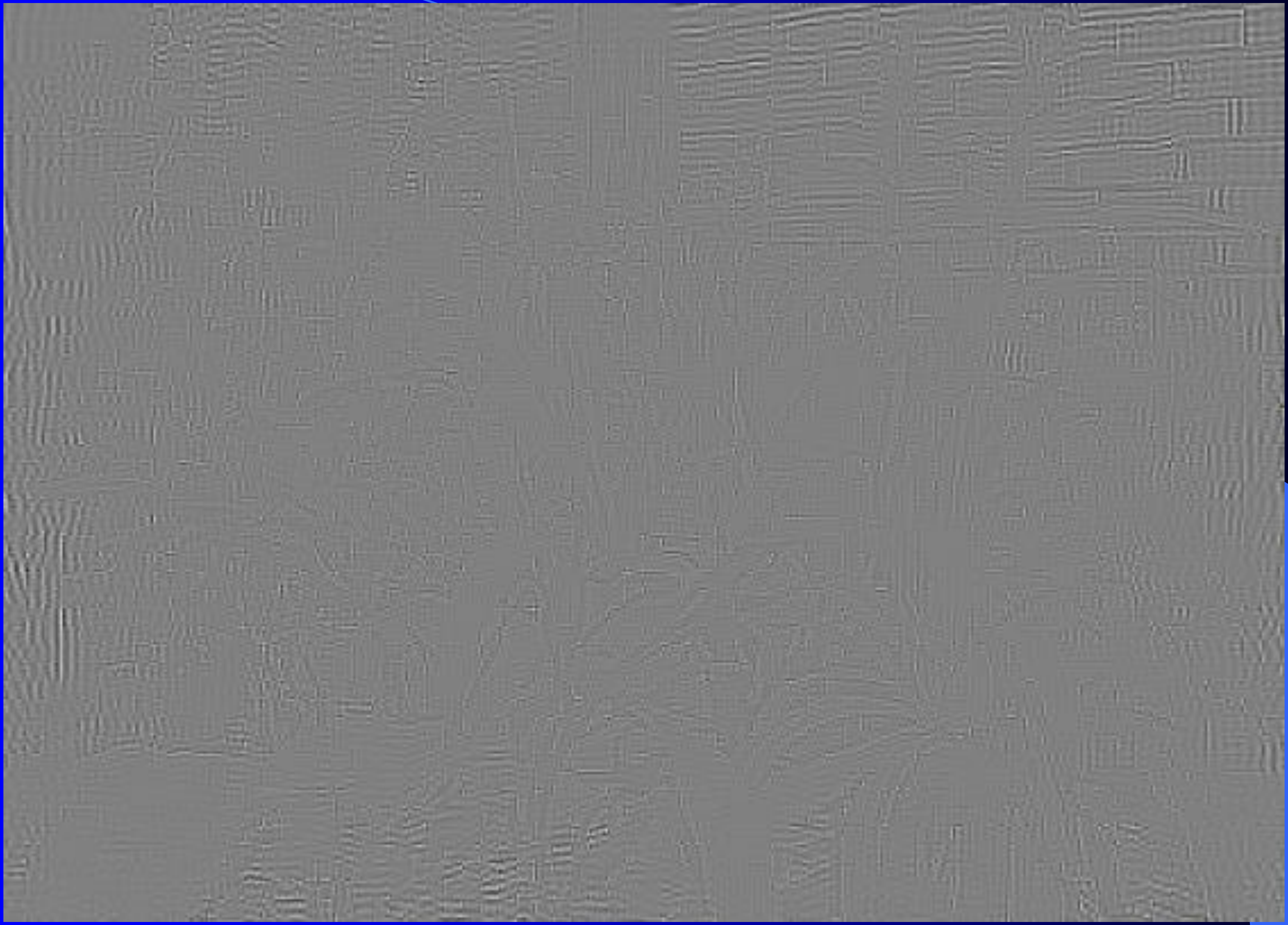


Image filtering and deblocking by projection filtering

Original lossy JPEG image

Enhanced image



Difference image (Subtracted high frequency information)

Zoomed in:

Original image

Enhanced image

Scanned image

Enhanced image

Zoomed in:

Scanned image

Enhanced image

Image matching

Information parameterization for image database retrieval

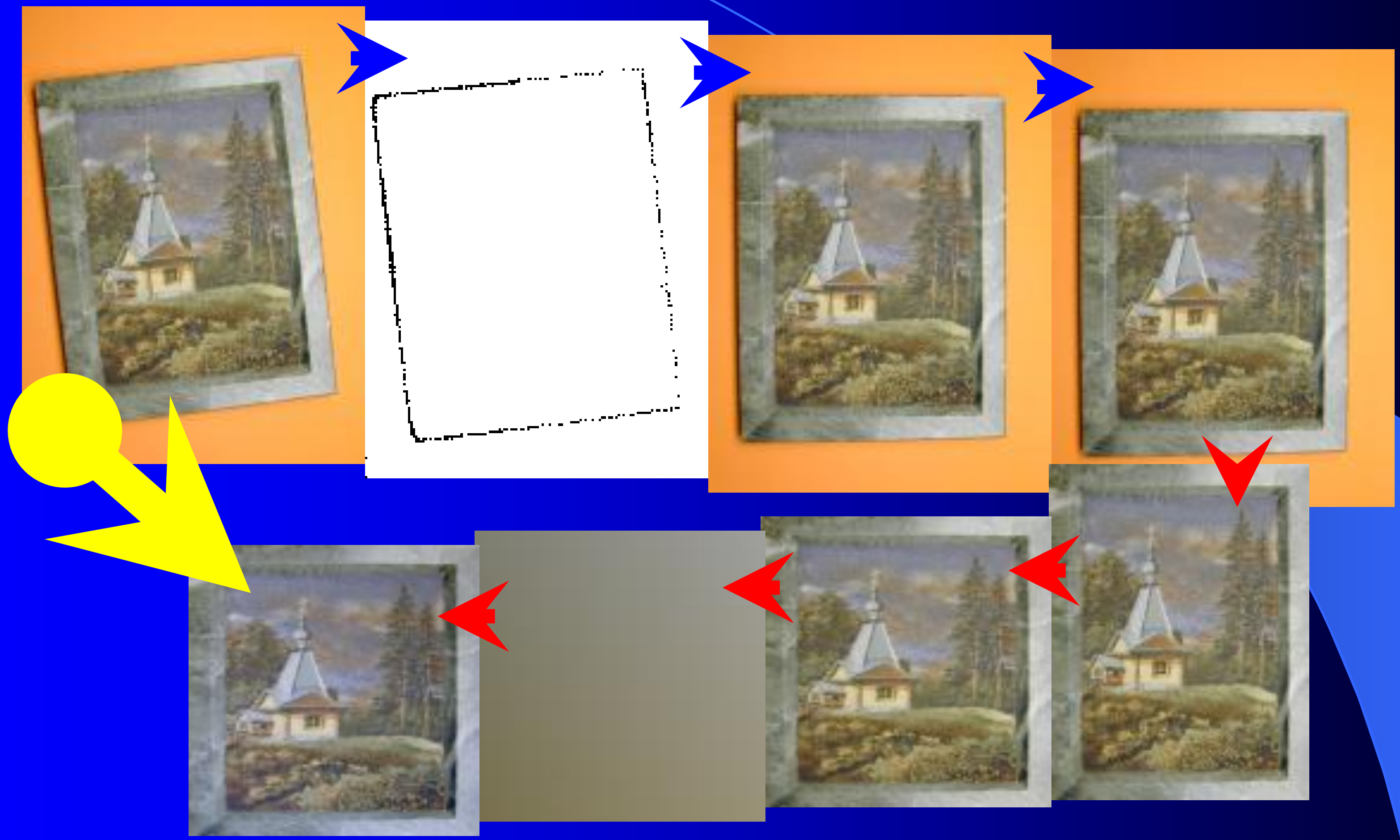


Image normalizing

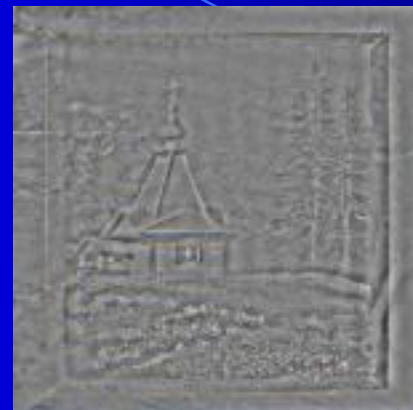
Graphical information parameterization

Parameterized image retrieval

Information parameterization for image database retrieval

Normalized image

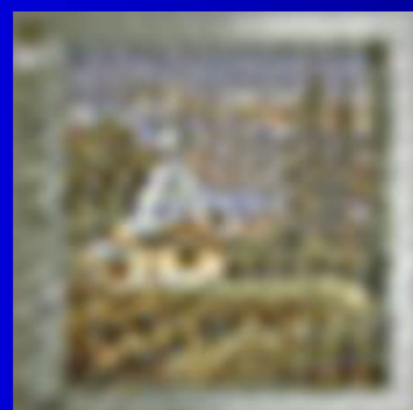
=



HF component

+

LF component



Recovered image with the recovered image plane

Algorithm

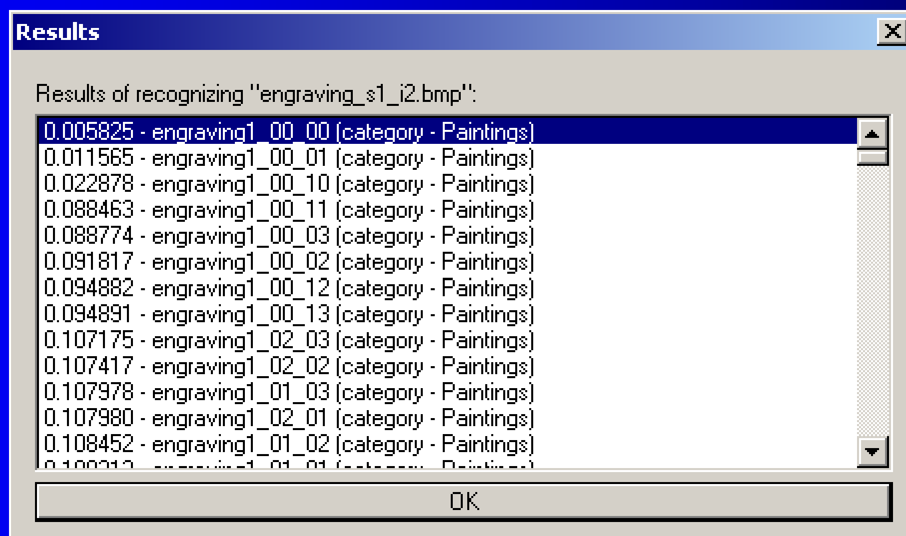
Image normalizing

Graphical information parameterization

Parameterized image retrieval

Information parameterization for image database retrieval

Database size	– 768 images (4.12Gb)
Initial images format	– 1600x1200x24bit (5.5Mb)
Normalized images format	– 512x512x24bit (0.75Mb)
Number of parameterization coefficients	– 32x32x3
Error rate	– <0.14%
Search time	– 4 sec. (for K7-750)



Algorithm

Image normalizing → Graphical information parameterization → Parameterized image retrieval


Hermite coder pro - Dscn0012
File
View
Image
Settings
Window
Help
Graphics:
Polyline
Coef
All image
Coef info
Pass:
X
X&Y
String #:
1
Number of functions (X):
32
Number of functions (Y):
24
Professional settings
Auto
Draw
Dscn0012
Dscn0009
Dscn0011
Dscn0010
Ready
Dscn0012.bmp
1600x1200x24b
Best fit
14:27:21

Image matching results

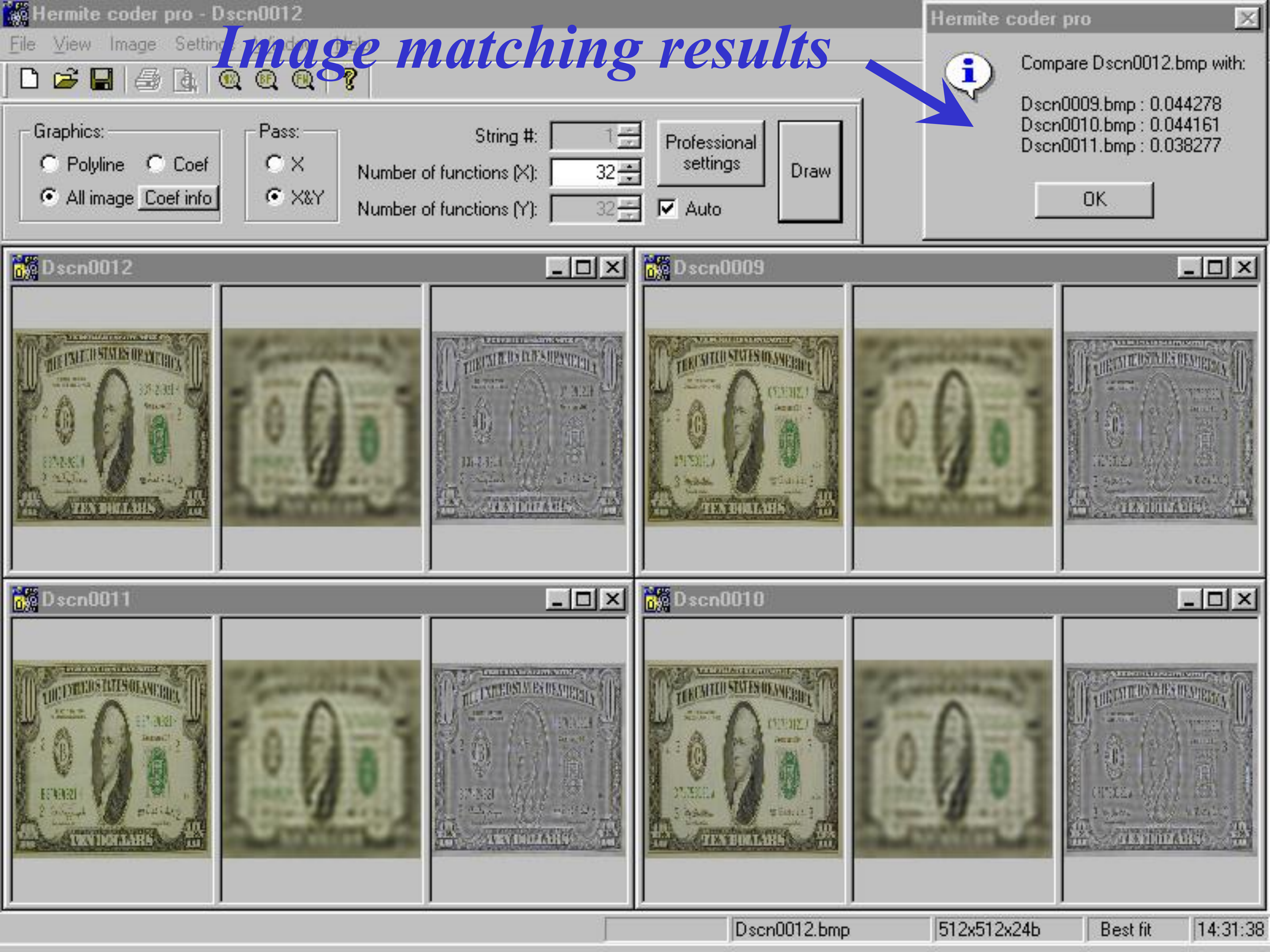
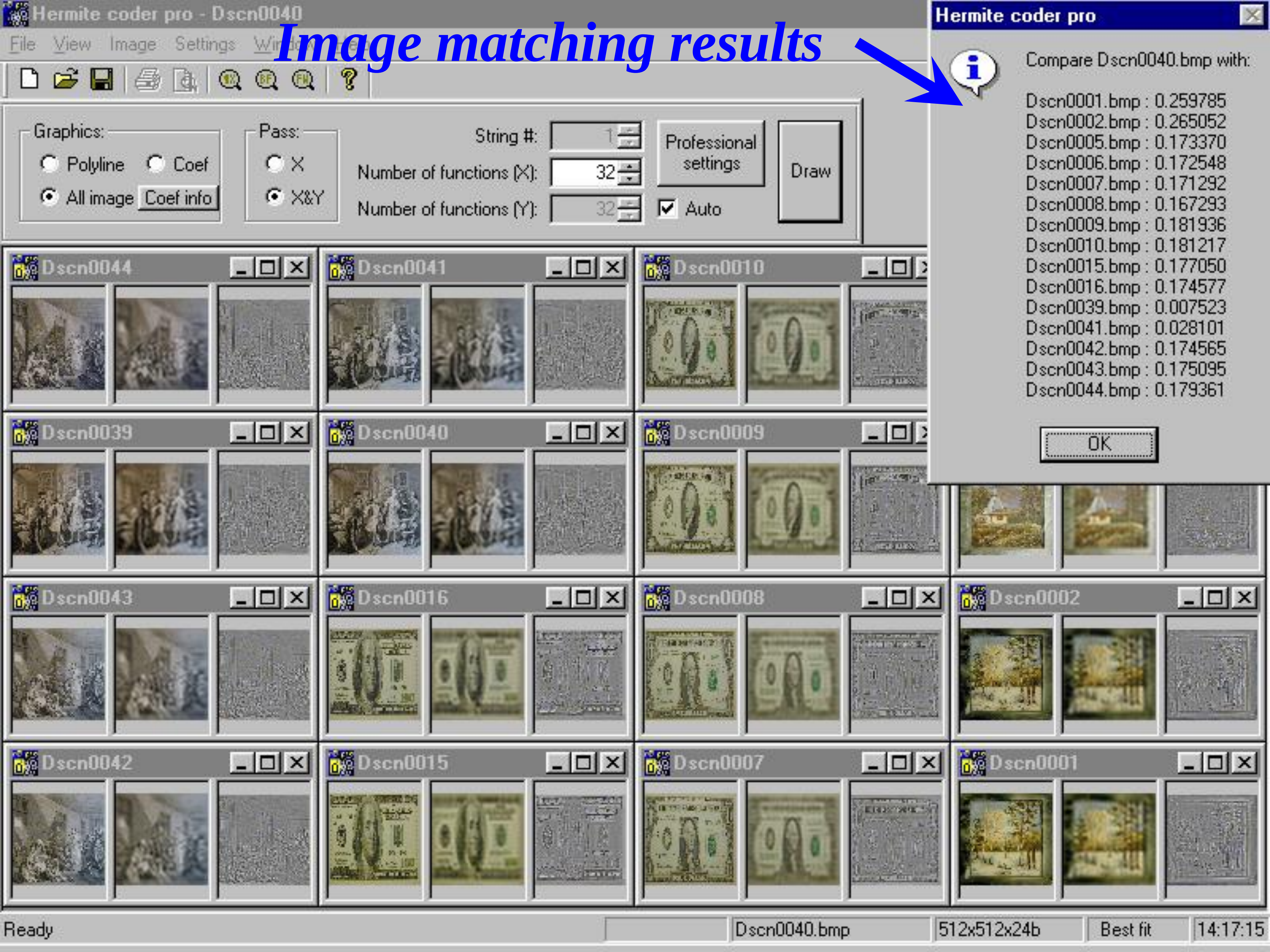
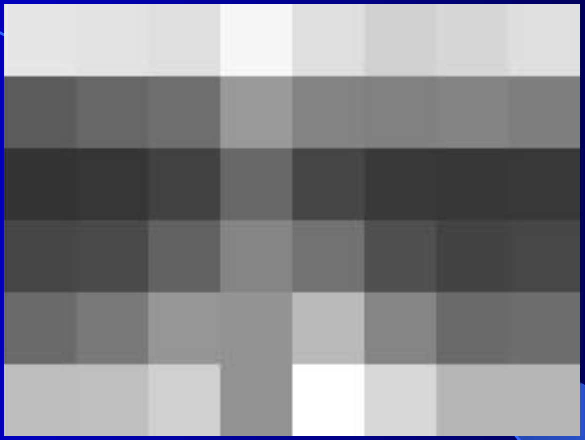
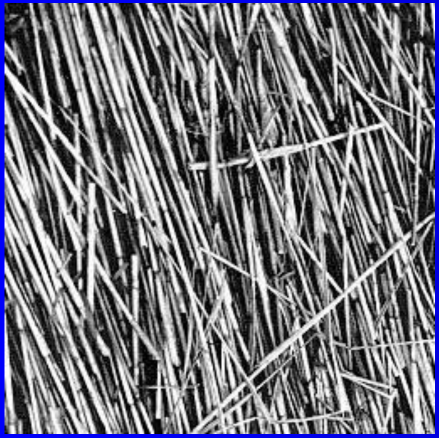
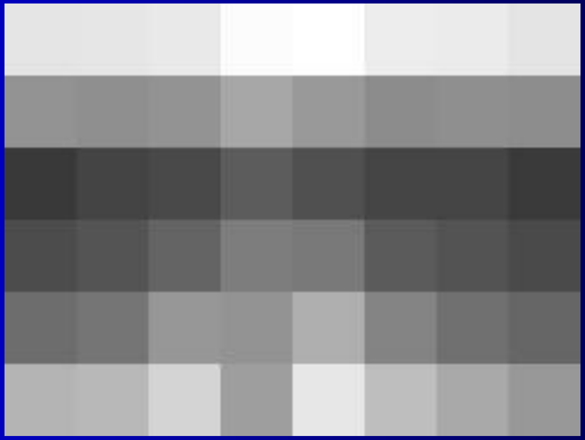


Image matching results



Texture matching

A method of obtaining the texture feature vectors

Input function

$$f(x) = \sum_{i=0}^{\infty} \alpha_i \cdot \Psi_i(x)$$

Fourier coefficients

$$\alpha_i = \int_{-\infty}^{\infty} \Psi_i(x) \cdot f(x) dx$$

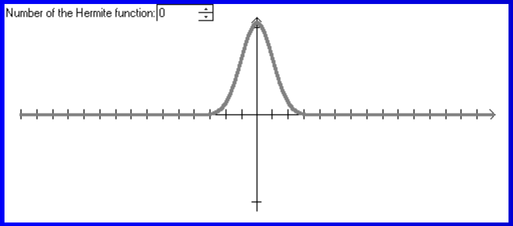
1-D to 2-D expansion

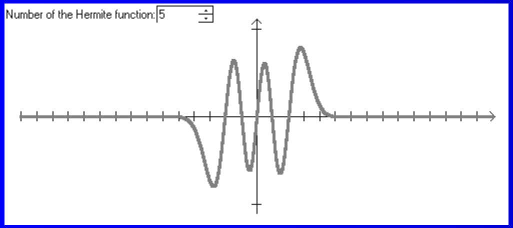
$$\psi_{n_1 n_2}(x, y) = \psi_{n_1}(x) \cdot \psi_{n_2}(y),$$

$$\psi_n(x, y) = \psi_n(x) \cdot 1$$

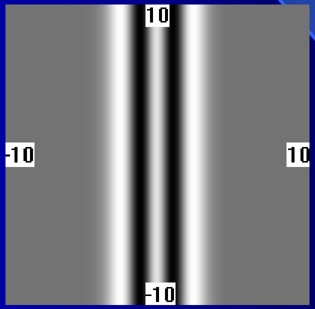
A method of obtaining the texture feature vectors

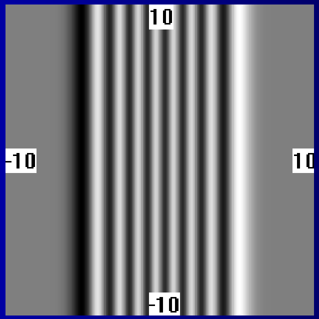
1-D Hermite functions:



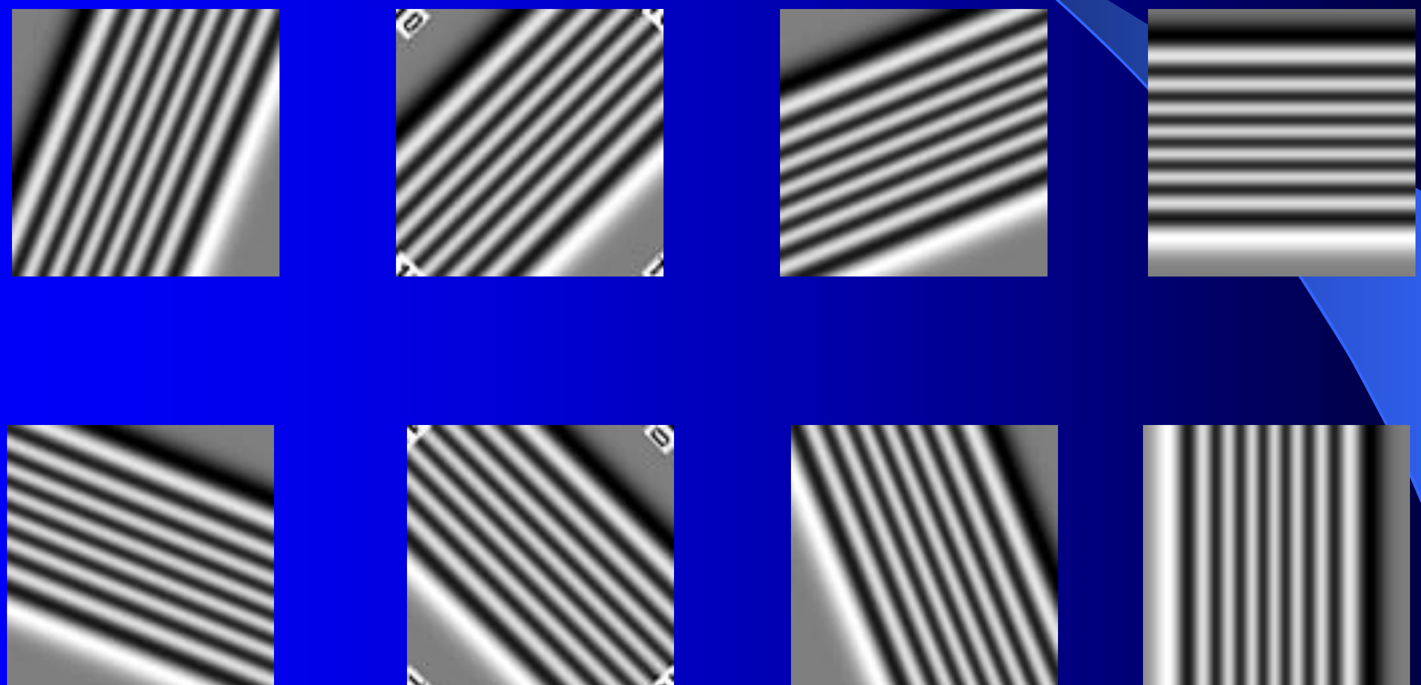


1-D to 2-D expanded Hermite functions:

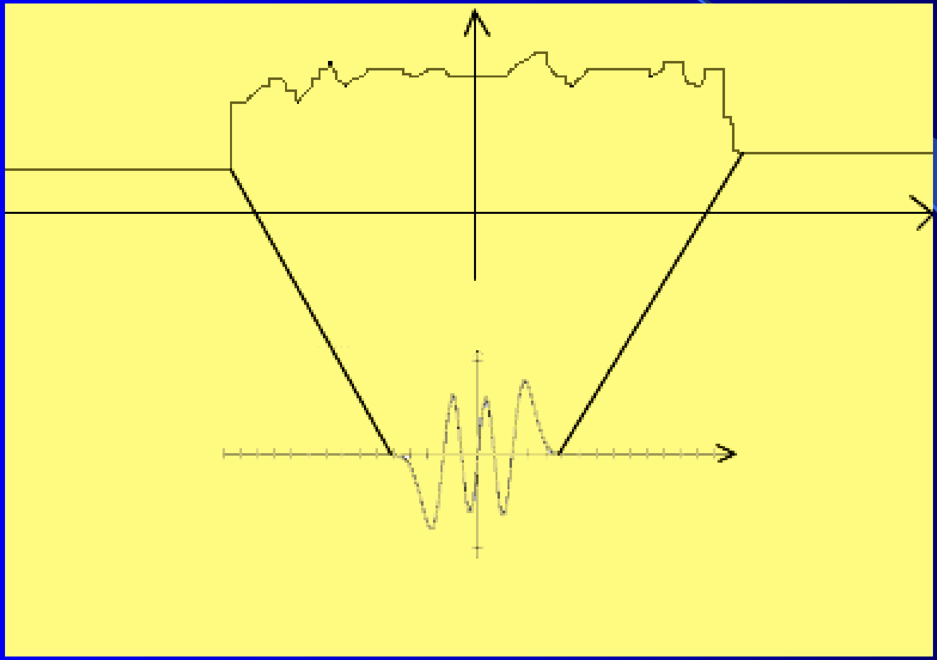




Orientations



Localization problem



Decomposition process is optimal, if localization segments of the input function and filtering functions are equal.

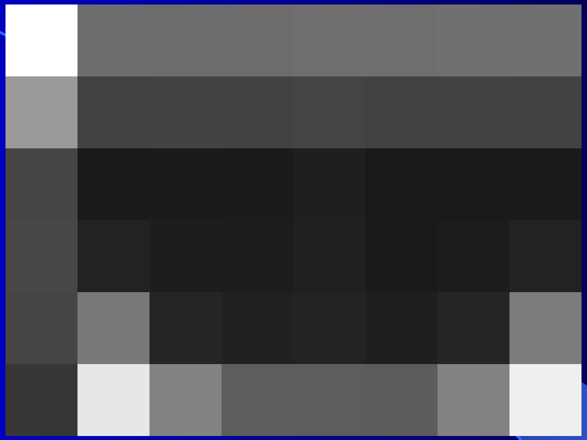
Feature vectors

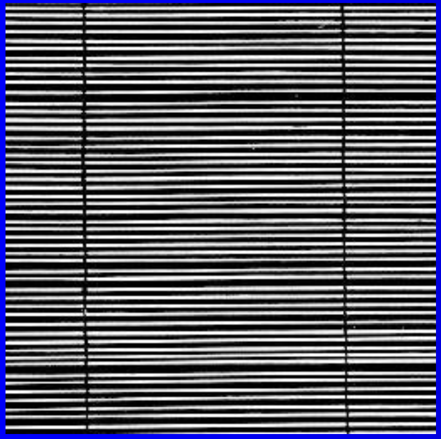
Standard coding

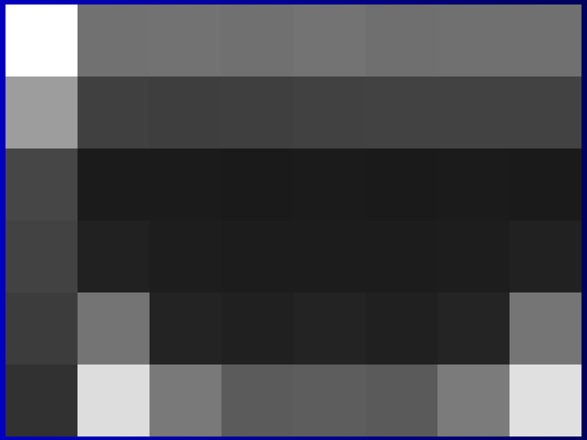
In this approach to get the feature vectors we consider the functions $\psi_n(x,y)$ where n1=0..64, and 6 energy coefficients are calculated as:

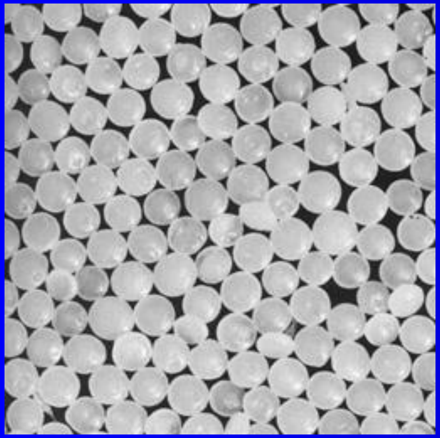
$$E_1 = (\alpha_0)^2 + (\alpha_1)^2 ,$$
$$E_2 = (\alpha_2)^2+(\alpha_3)^2 + (\alpha_4)^2,$$
$$E_3= (\alpha_5)^2 +(\alpha_6)^2+(\alpha_7)^2+(\alpha_8)^2,$$
$$\ldots$$
$$E_6 = (\alpha_{33})^2+(\alpha_{34})^2 + \ldots + (\alpha_{63})^2 +(\alpha_{64})^2 ,$$

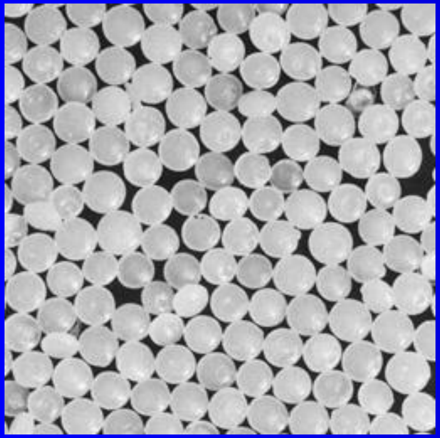
f(x,y) is the source image.











Feature vectors

Hierarchical coding

$$E_1 = (\alpha_0^{(1)})^2 + (\alpha_1^{(1)})^2,$$
$$E_2 = (\alpha_0^{(2)})^2 + (\alpha_1^{(2)})^2 + (\alpha_2^{(2)})^2 + (\alpha_3^{(2)})^2,$$
$$\ldots$$
$$E_6 = (\alpha_0^{(6)})^2 + (\alpha_1^{(6)})^2 + \ldots + (\alpha_{62}^{(6)})^2 + (\alpha_{63}^{(6)})^2,$$

$$\alpha_i^{(1)} = \frac{1}{\sqrt{A_j}} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \psi_i(x,y) \cdot f(x,y)\, dx$$

$$\alpha_i^{(j)} = \frac{1}{\sqrt{A_j}} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \psi_i(x,y) \cdot \left(f(x,y) - \sum_{l=2}^{j} f^{(l-1)}(x,y)\right) dx,\ j > 1$$

f(x,y) is the source image.

Feature vectors

Hierarchical coding without subtractions

$$E_1 = (\alpha_0^{(1)})^2 + (\alpha_1^{(1)})^2,$$

$$E_2 = (\alpha_0^{(2)})^2 + (\alpha_1^{(2)})^2 + (\alpha_2^{(2)})^2 + (\alpha_3^{(2)})^2,$$

$$\ldots$$

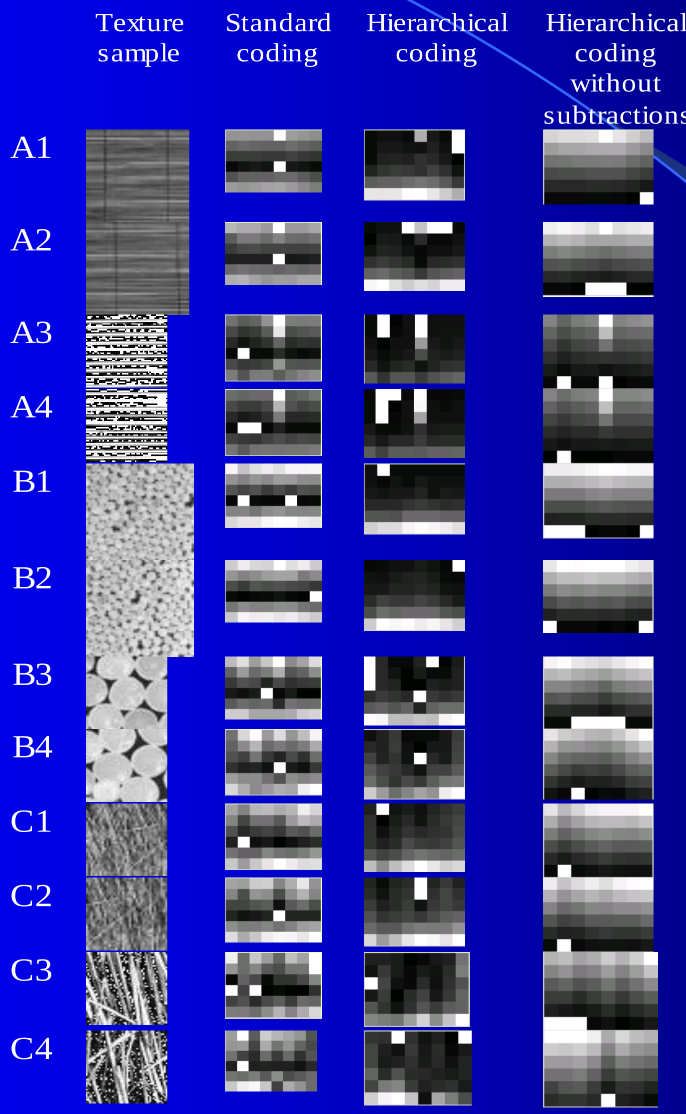
$$E_6 = (\alpha_0^{(6)})^2 + (\alpha_1^{(6)})^2 + \ldots + (\alpha_{62}^{(6)})^2 + (\alpha_{63}^{(6)})^2,$$

$$\alpha_i^{(1)} = \frac{1}{\sqrt{A_j}} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \psi_i(x,y) \cdot f(x,y)\,dx$$

$$\alpha_i^{(j)} = \frac{1}{\sqrt{A_j}} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} \psi_i(x,y) \cdot f(x,y)\,dx,\ j > 1$$

f(x,y) is the source image.

Feature vectors



Feature vectors

a	*b*	Standard coding	Hierarchical coding	Hierarchical coding without subtractions
a1	a2	0.004505	0.005349	0.003739
b1	b2	0.009905	0.008160	0.007742
c1	c2	0.017778	0.011848	0.009946
a3	a4	0.008807	0.006047	0.002422
b3	b4	0.020075	0.010667	0.006275
c3	c4	0.128322	0.101591	0.080176
a1	b1	0.488420	0.492238	0.491653
b1	c1	0.315644	0.304597	0.305442

Image segmentation task: Brodatz textures

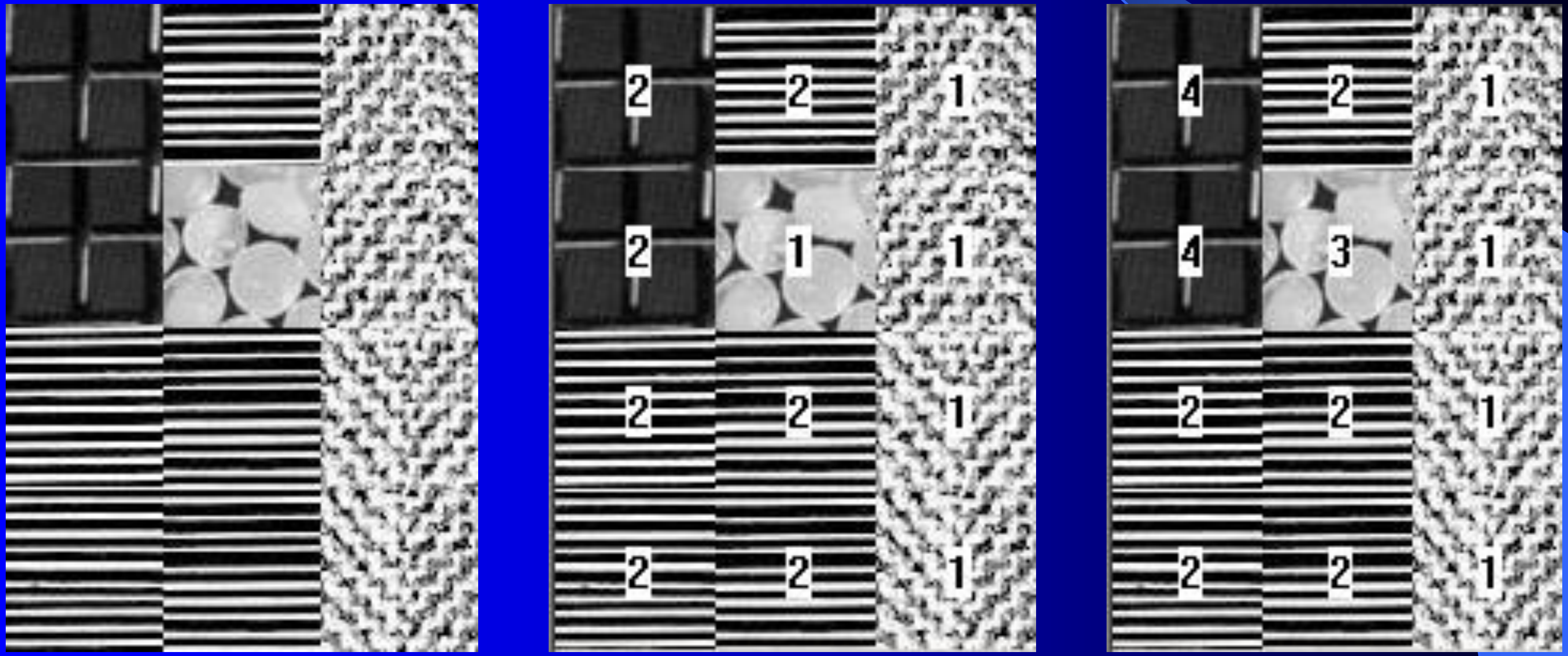


Image segmentation task

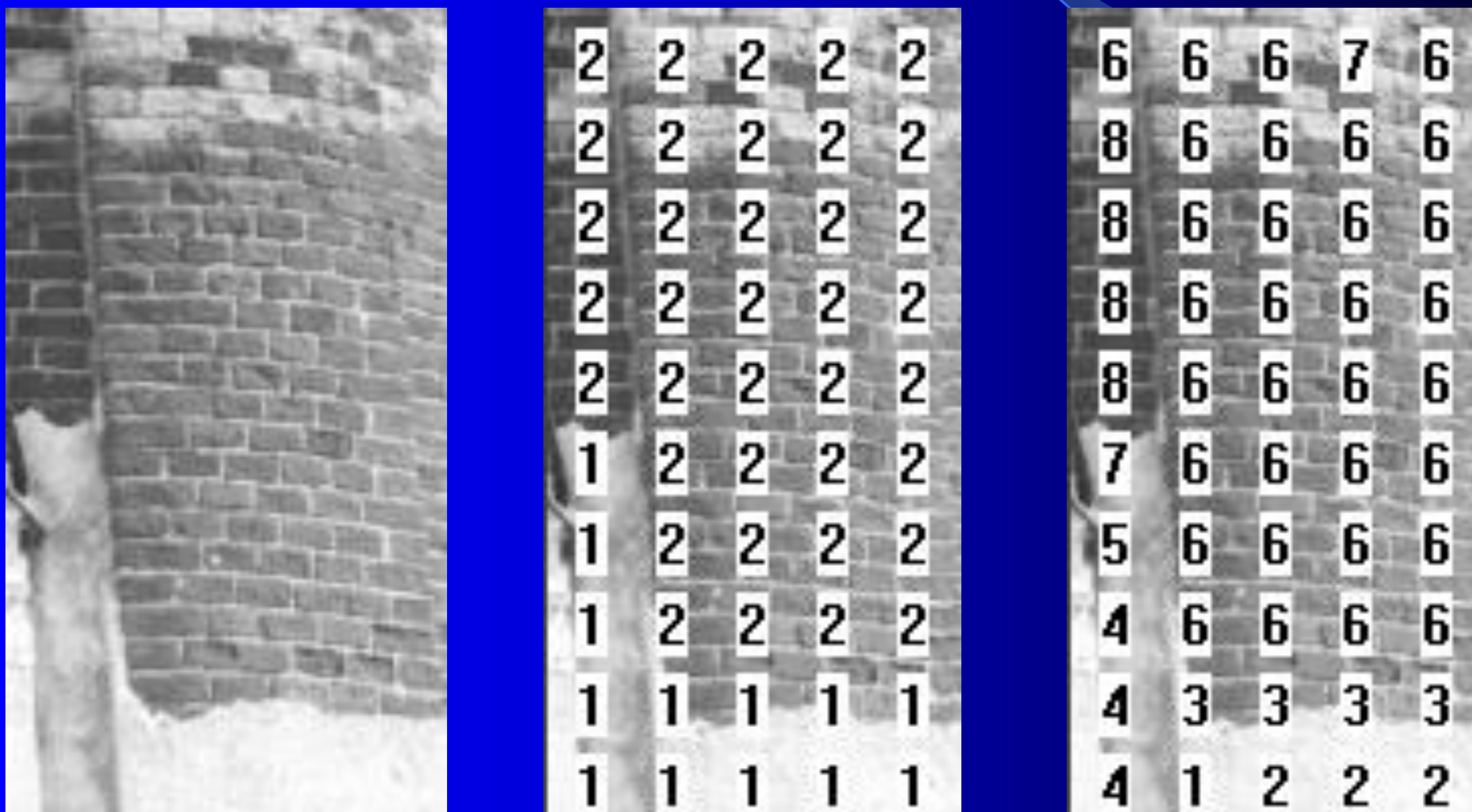
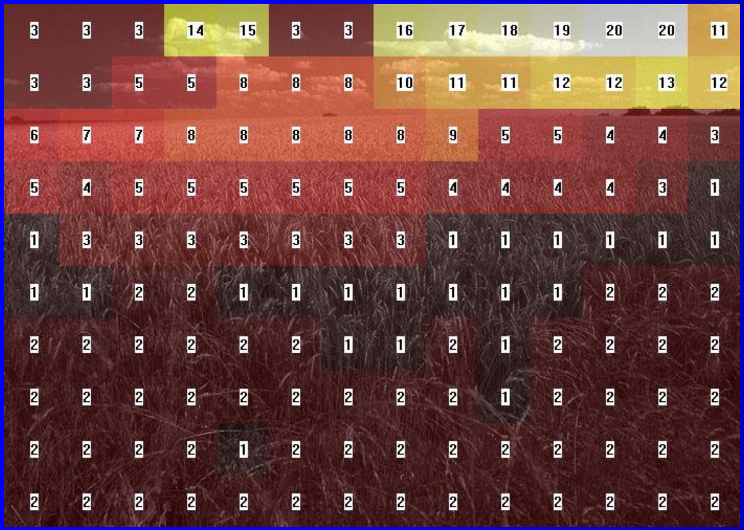


Image segmentation task



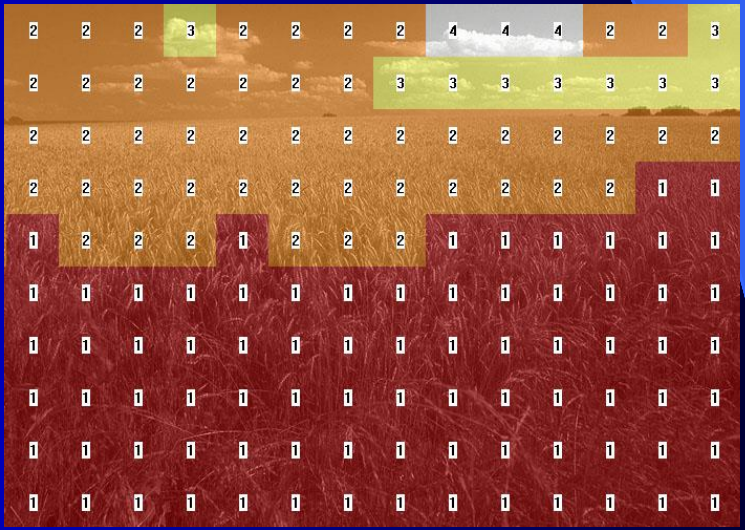
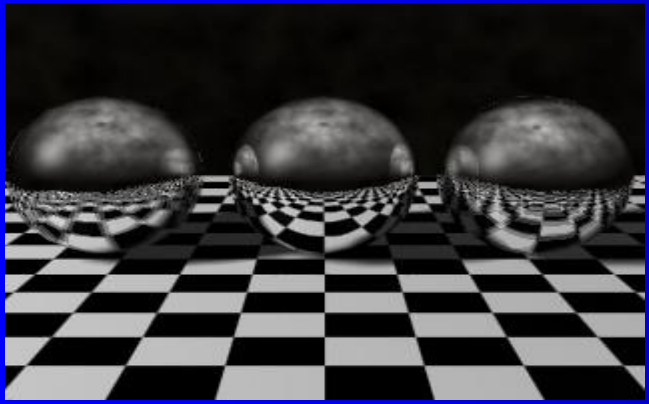
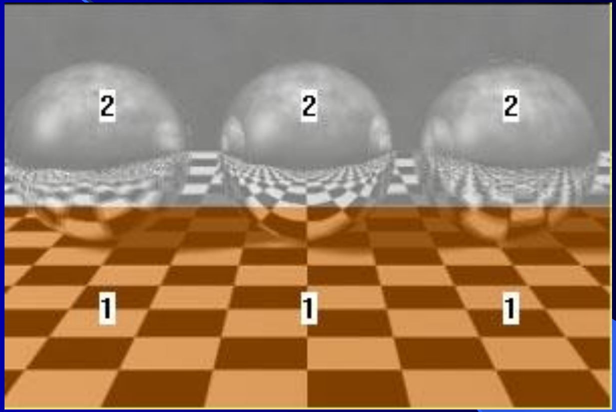
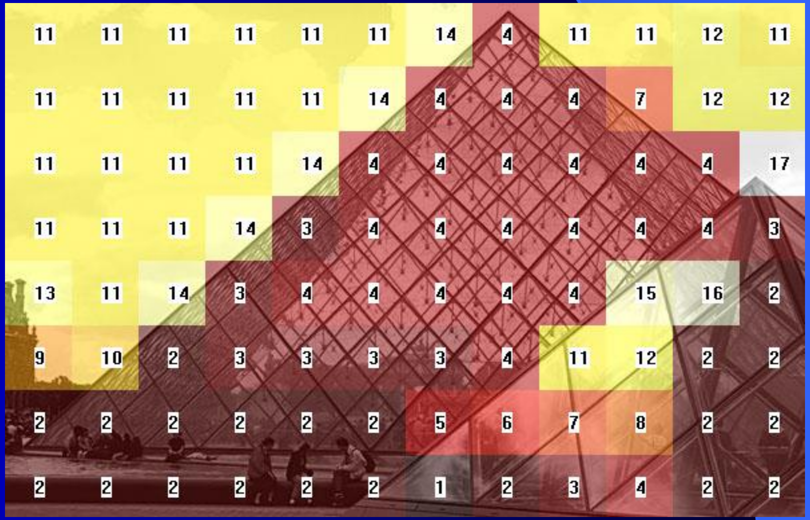


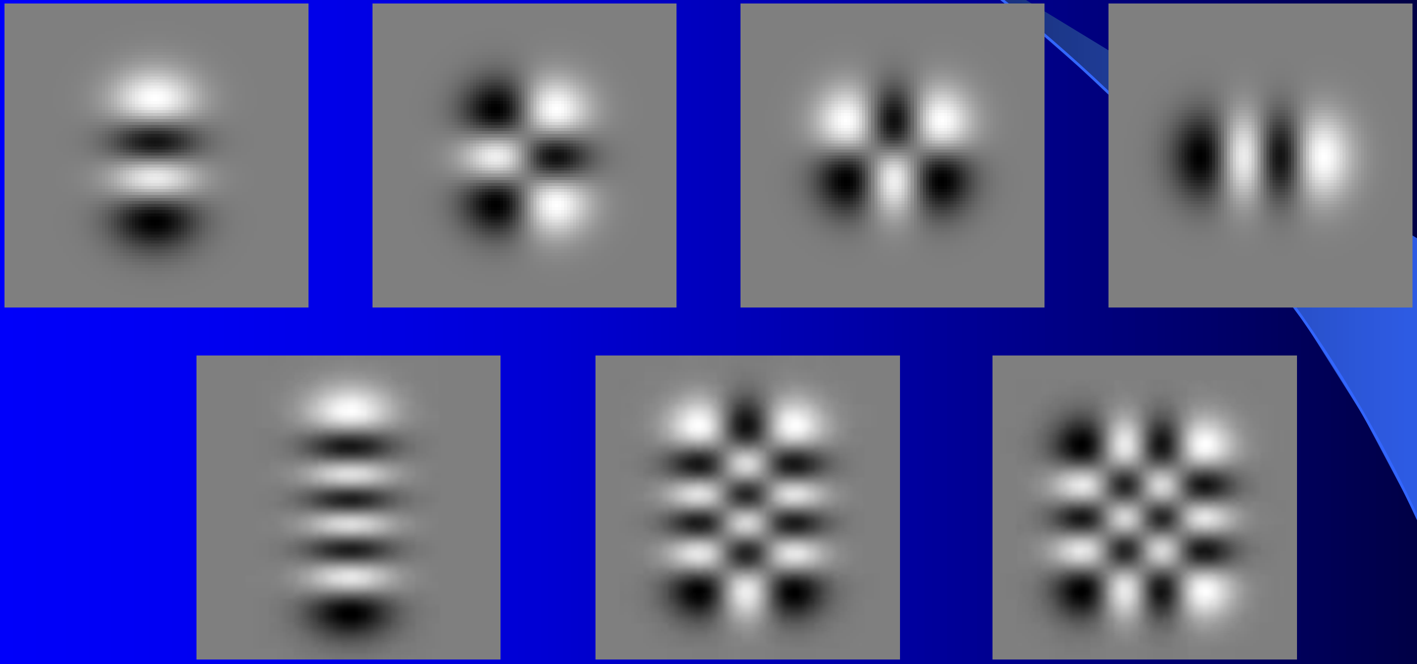
Image segmentation task





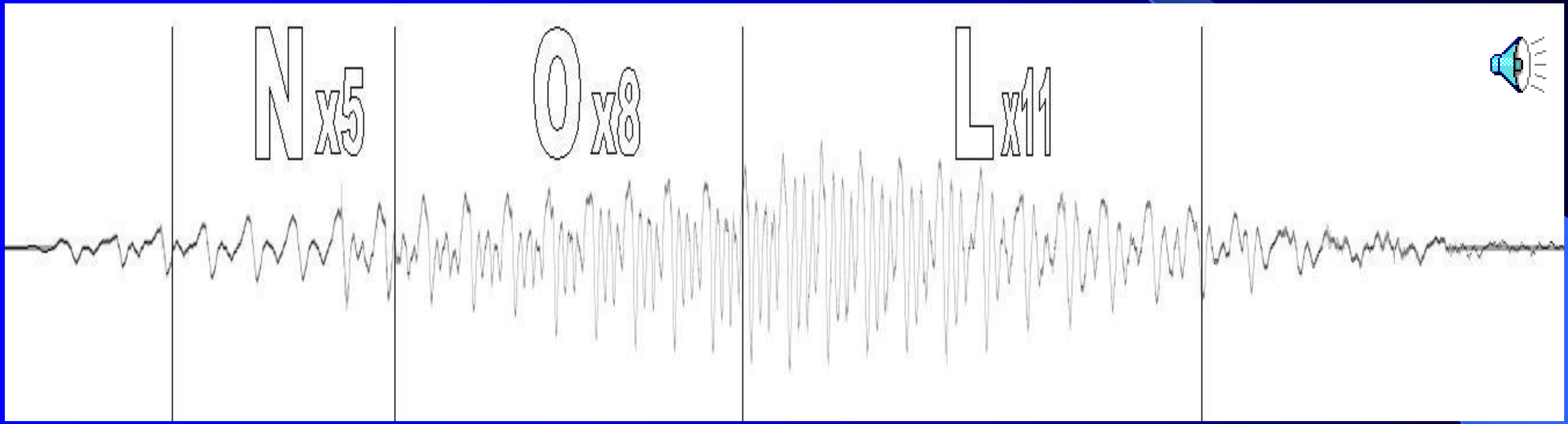


Texture parameterization using 2-D Hermite functions

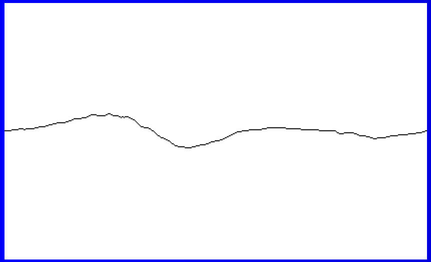
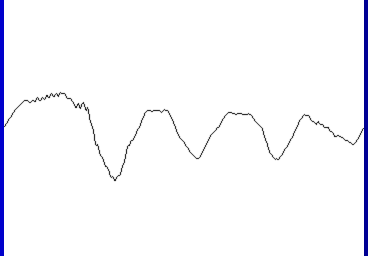
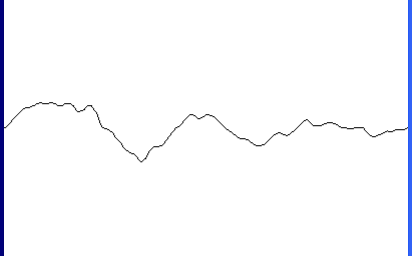


$$\psi_{0,3}(x,y),\psi_{1,2}(x,y),\psi_{2,1}(x,y),\psi_{3,0}(x,y),\psi_{0,7}(x,y),\psi_{2,5}(x,y),\psi_{3,4}(x,y)$$

Low-level methods for audio signal processing



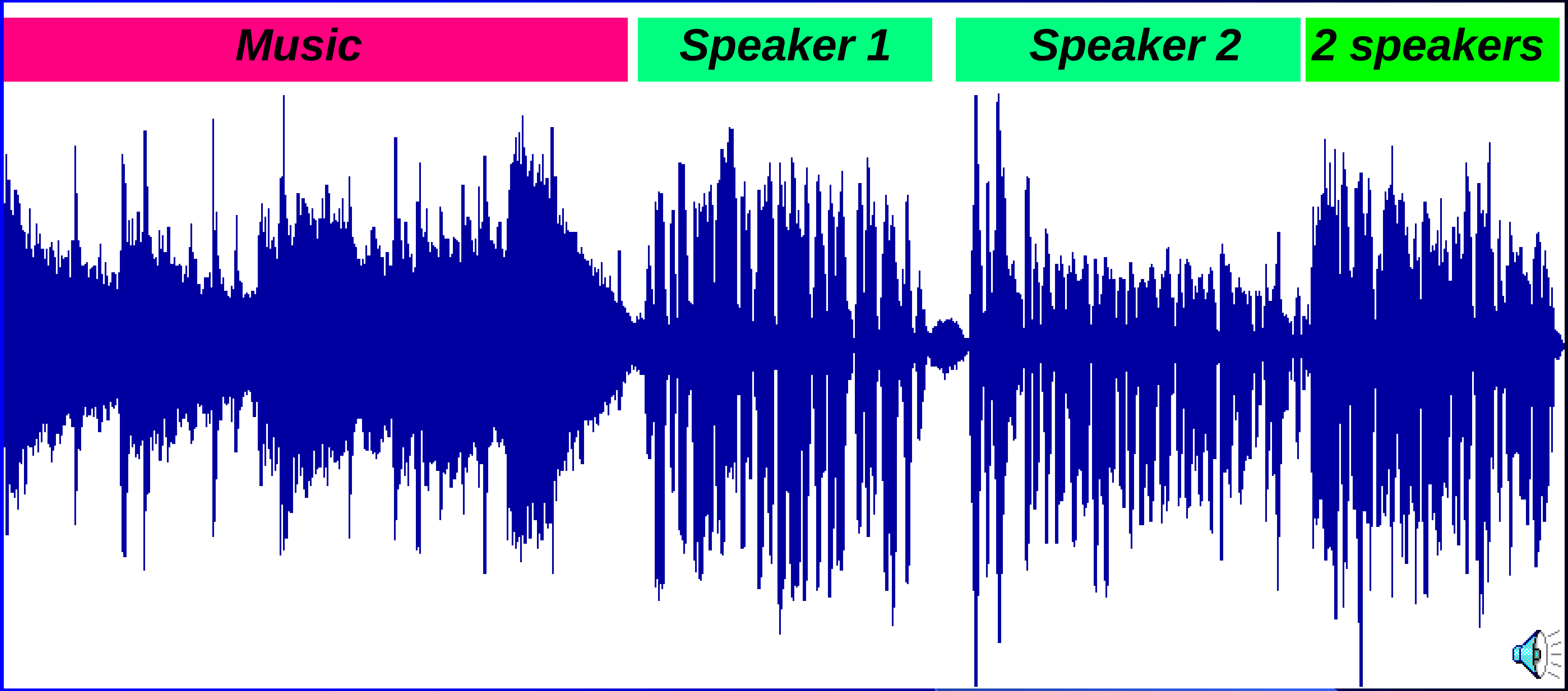
Quasiperiod's waveforms

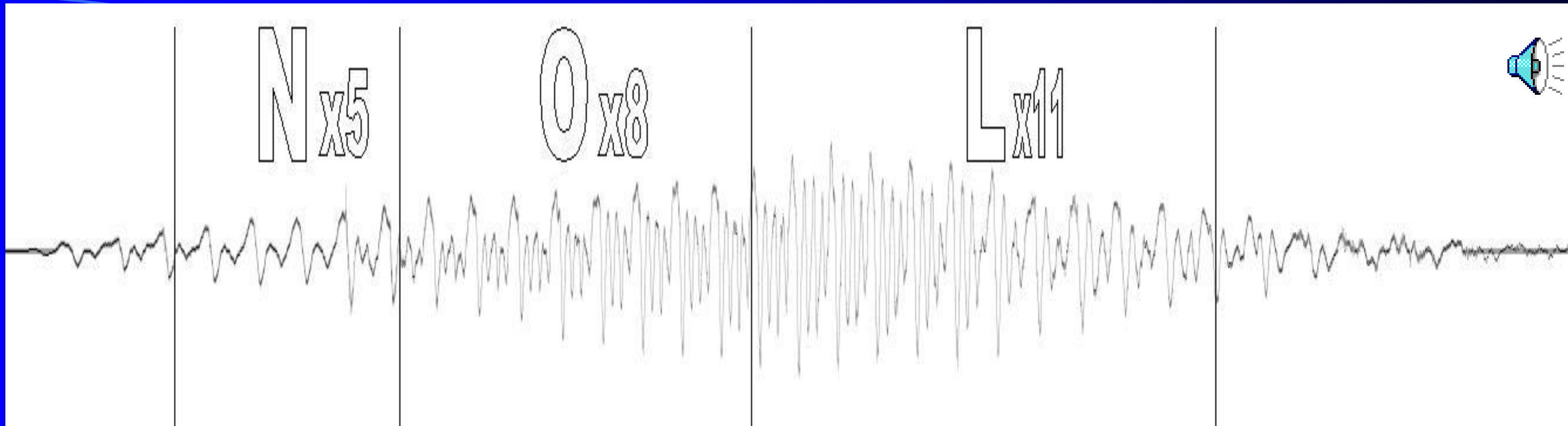
N  O  L 

Areas of Hermite transform application:

- Signal filtering
- Speaker indexing
- Speaker recognition using database
- Source separation

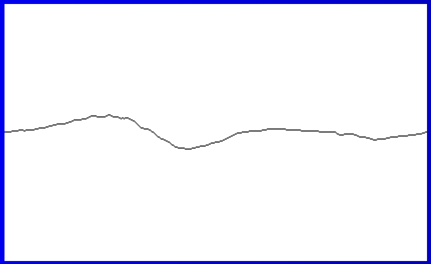
Audio sample

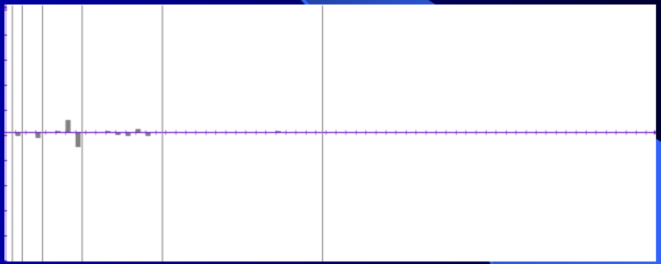




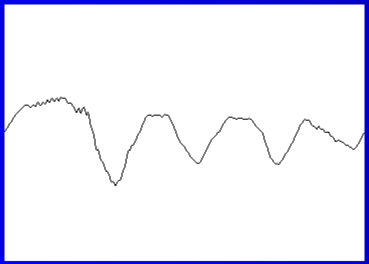
Quasiperiod waveform **Hermite histogram**

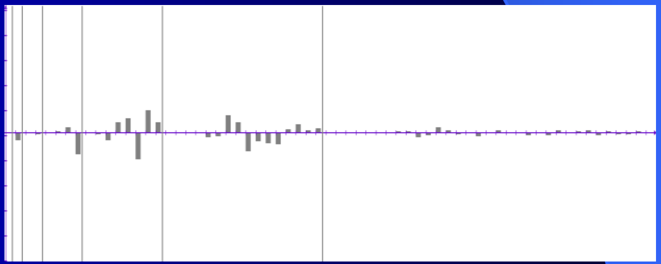
N



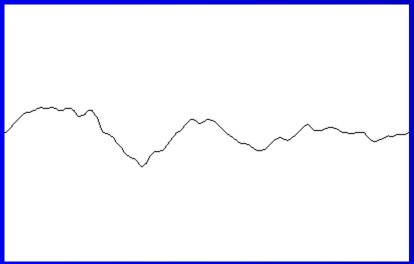


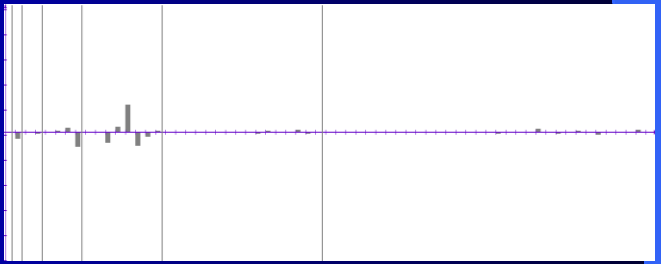
O



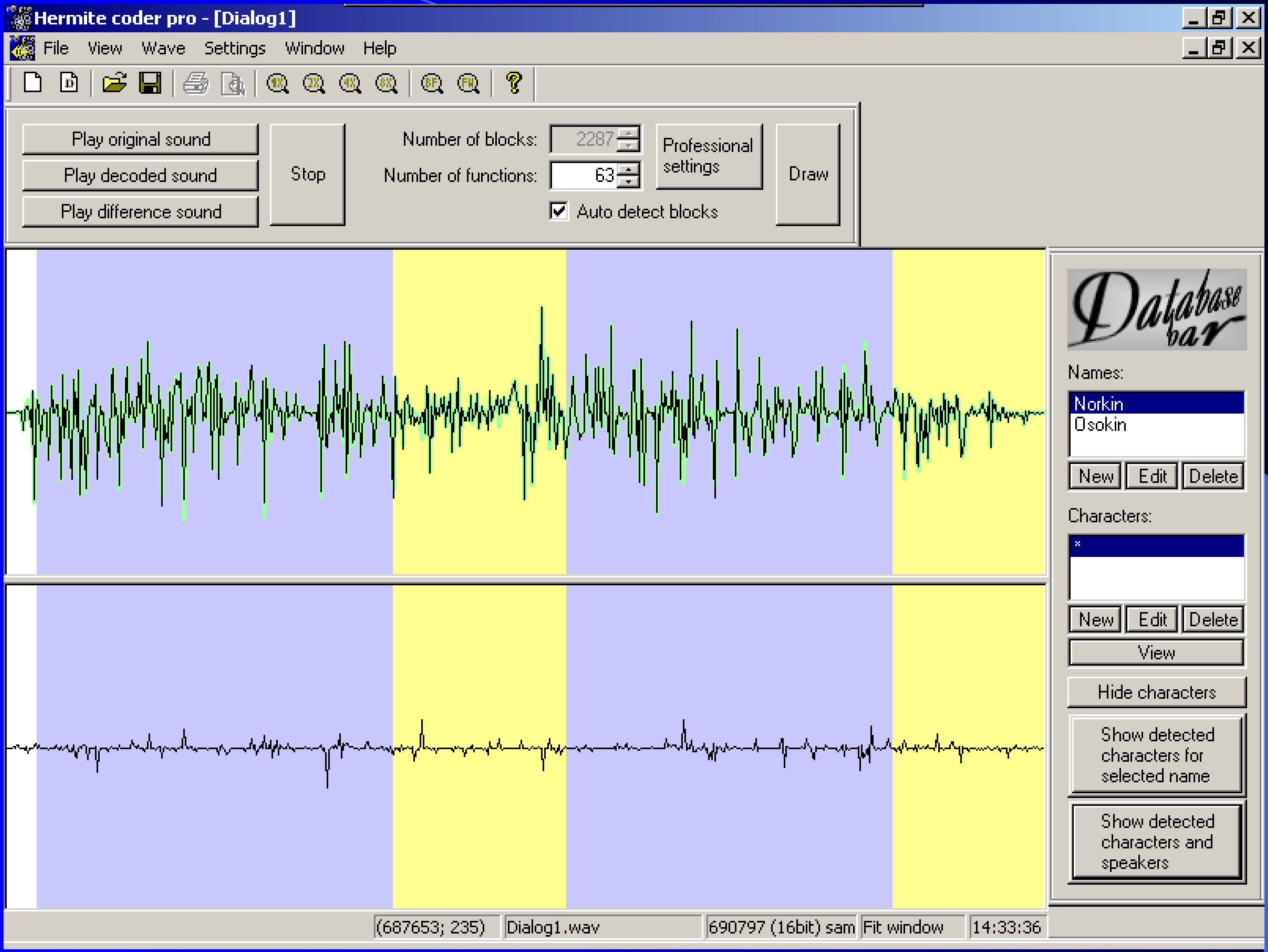


L

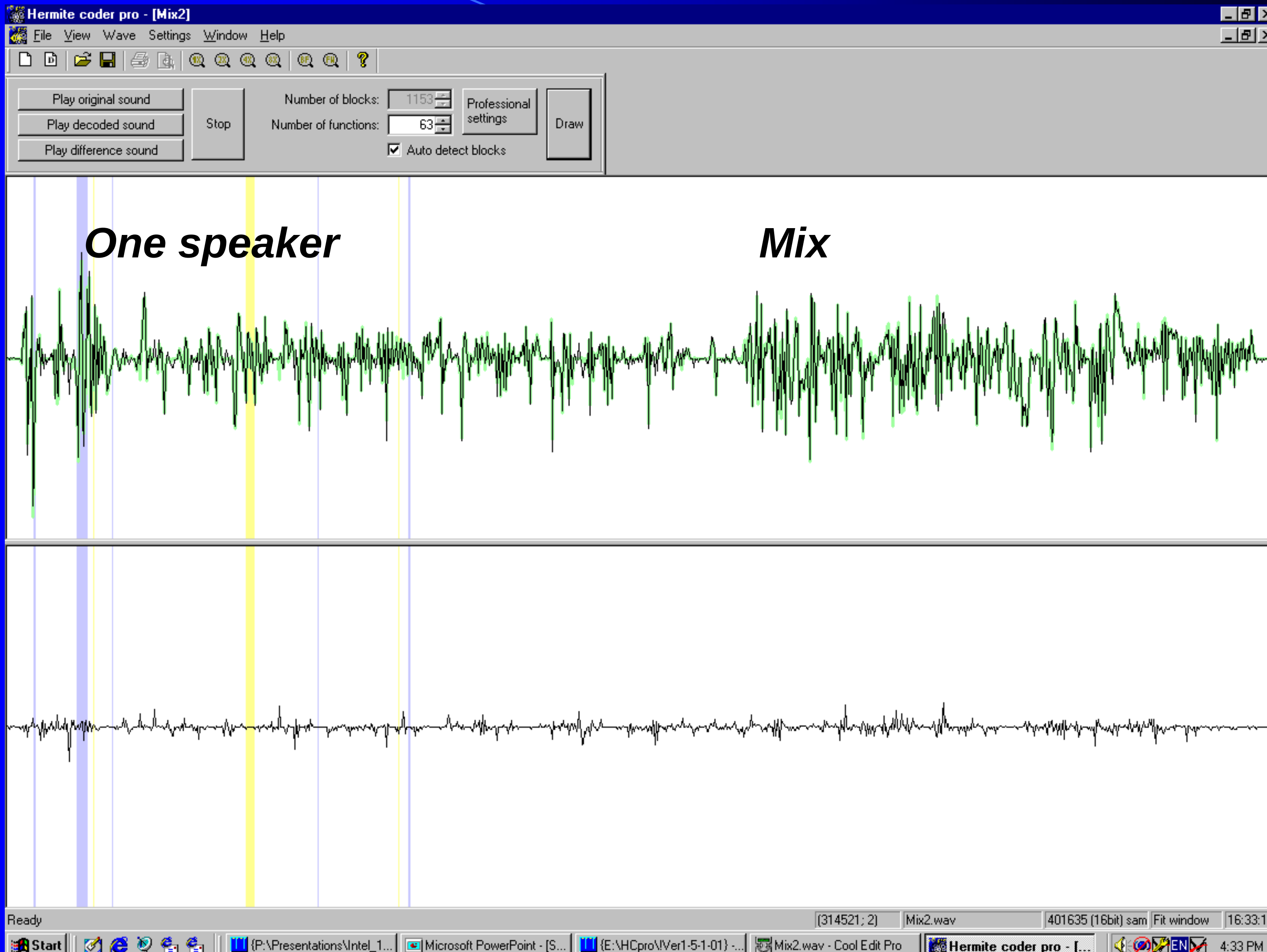




Speaker indexing

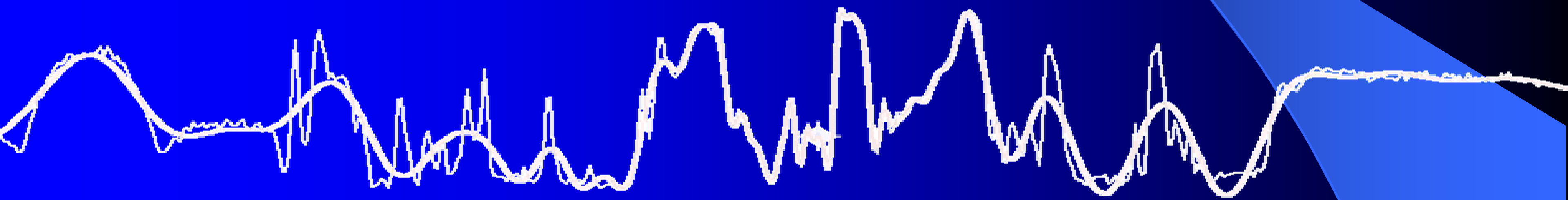


Mix detection



Faculty of Computational Mathematics and Cybernetics
Moscow State University

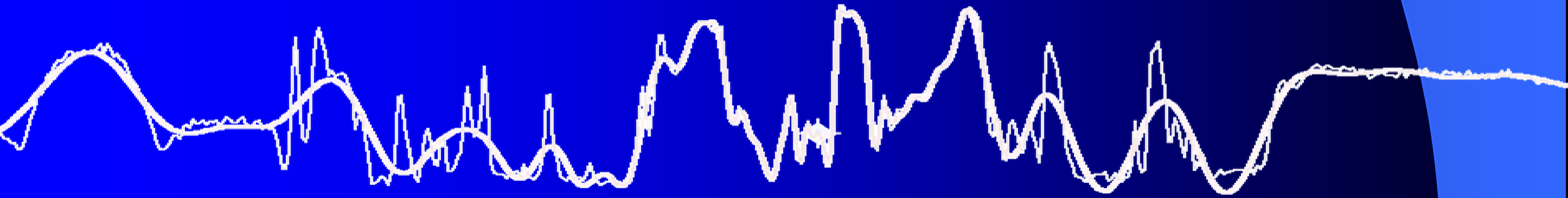
Hermite Foveation



A foveated image is a non-uniform resolution image whose resolution is highest at a point (fovea), but falls off away from the fovea.

$$(Tf)(x) = \int_{-\infty}^{\infty} k(x,t) f(t) dt$$

$$k(x,t) = \frac{1}{\alpha|x-\gamma|+\beta} g\left(\frac{t-x}{\alpha|x-\gamma|+\beta}\right)$$

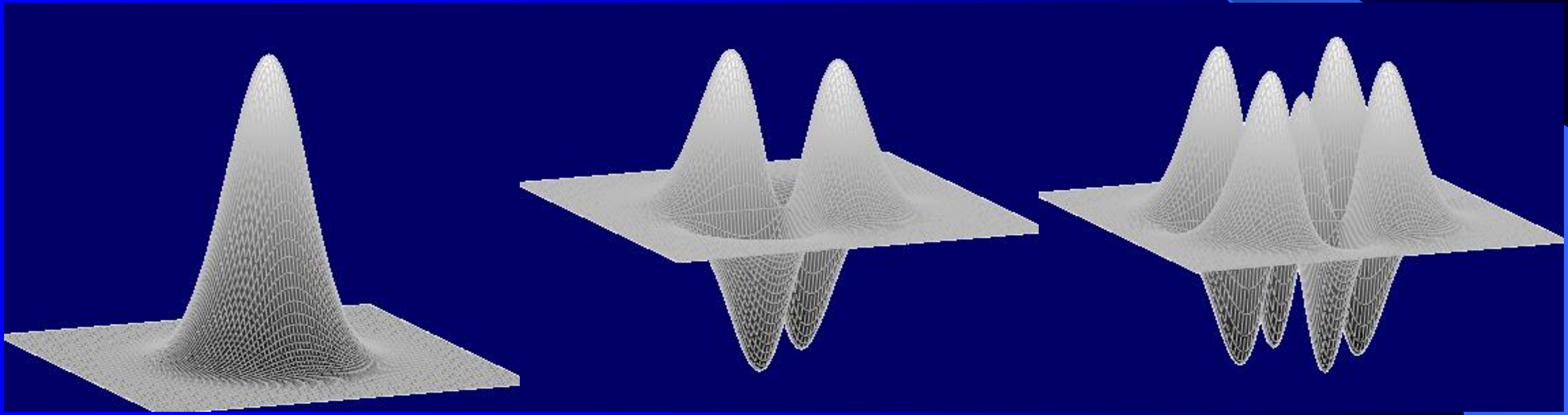


For foveation we used eigenfunctions of the Fourier transform (2D Hermite functions ψ_{nm}).

$$F(\psi_{nm}) = i^{n+m}\psi_{nm}$$

$$\psi_{nm}(x,y) = \frac{(-1)^{n+m} e^{x^2/2+y^2/2}}{\sqrt{2^{n+m} n! m! \pi}} \cdot \frac{d^n(e^{-x^2})}{dx^n} \cdot \frac{d^m(e^{-y^2})}{dy^m}$$

The graphs of the 2D Hermite functions look like the following:

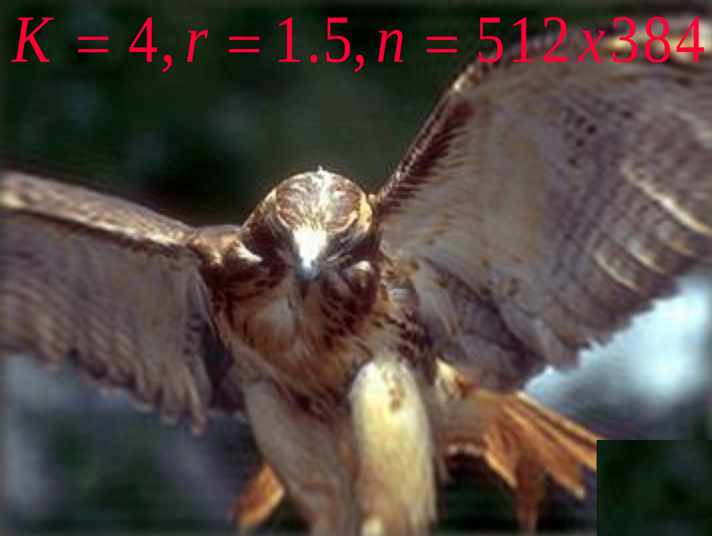


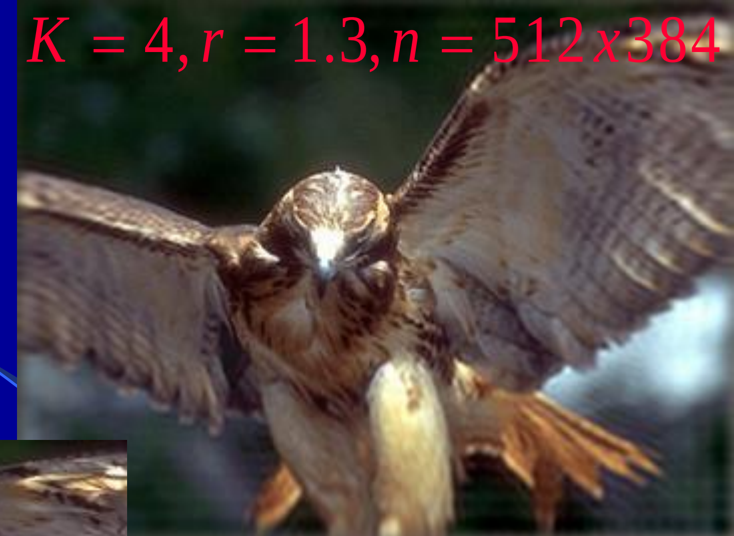
$\psi_{0,0}(x, y)$ $\psi_{1,1}(x, y)$ $\psi_{2,2}(x, y)$

The kernel for Hermite foveation was defined as:

$$k(x,t)=\sum_{i=0}^{\frac{n}{K}-1}\psi_i\left(A_{\frac{n}{K}-1}\frac{2x-w+1}{w}\right)\psi_i\left(A_{\frac{n}{K}-1}\frac{2t-w+1}{w}\right)+$$

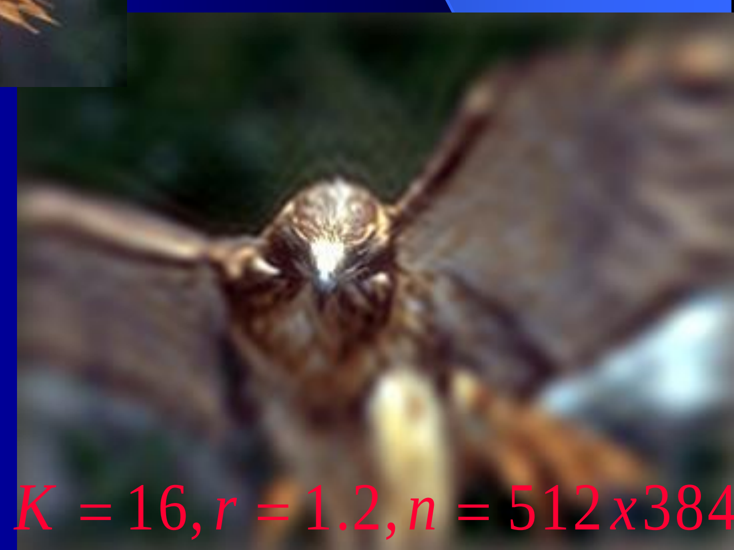
$$+\sum_{j=1}^{K-1}\left(\begin{array}{l}\max\left(\min\left(\frac{r}{r-1}\left(1-\frac{2r^j\left|\gamma-x\right|}{w}\right),1\right),0\right)\cdot\\ \sum_{i=0}^{\frac{n}{K}-1}\psi_i\left(A_{\frac{n}{K}-1}\frac{2x-w+1}{wr^j}\right)\psi_i\left(A_{\frac{n}{K}-1}\frac{2t-w+1}{wr^j}\right)\end{array}\right)$$

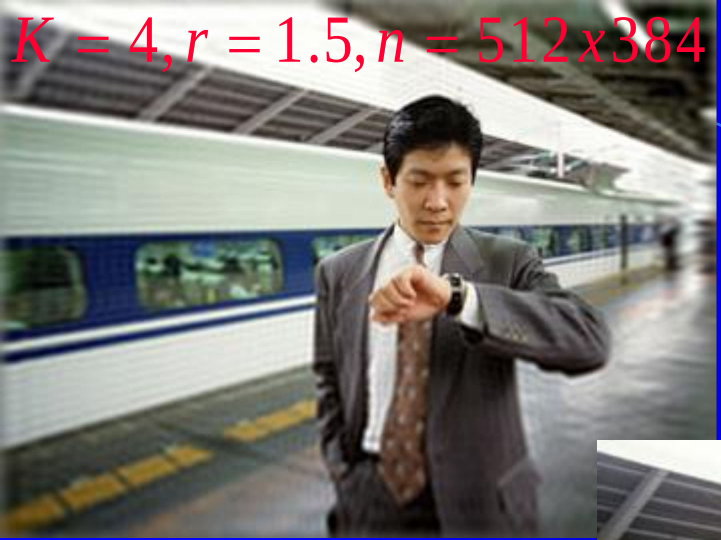

$K = 4, r = 1.5, n = 512x384$

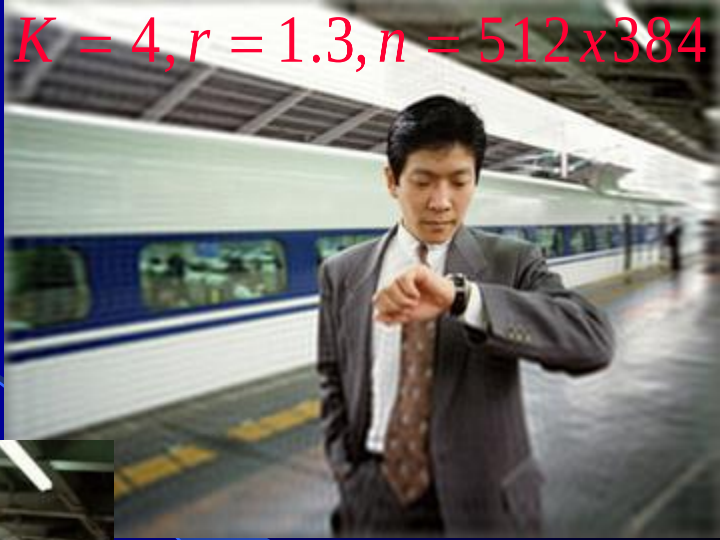

$K = 4, r = 1.3, n = 512x384$


Original image


$K = 8, r = 1.3, n = 512x384$

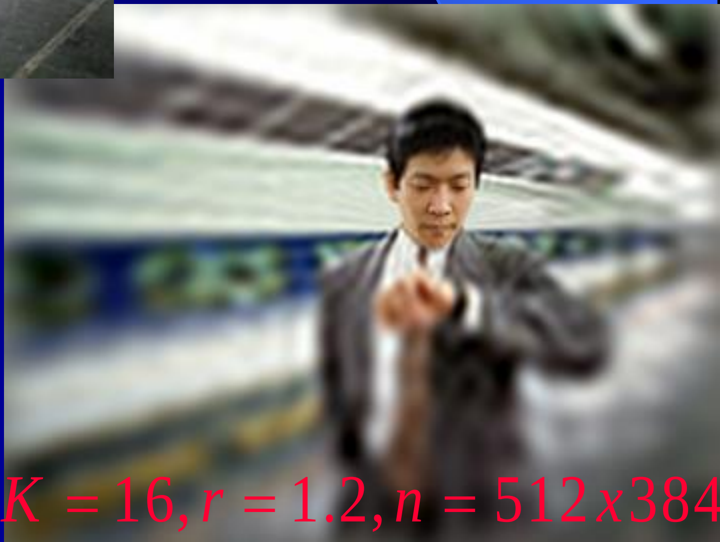

$K = 16, r = 1.2, n = 512x384$

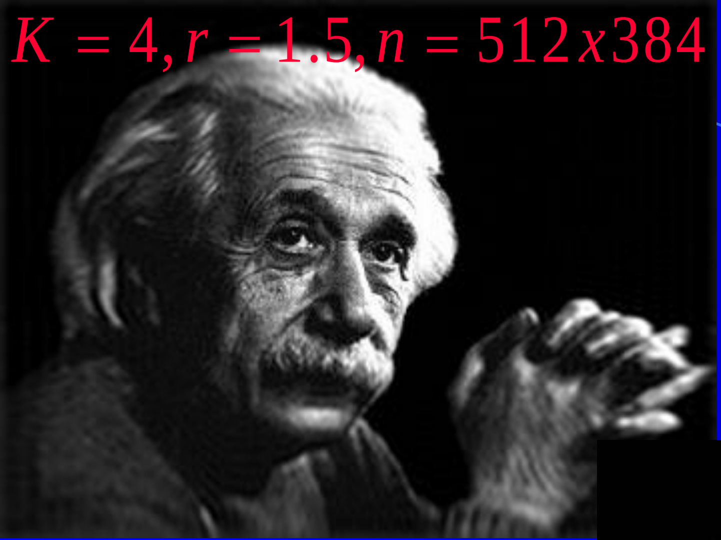

$K = 4, r = 1.5, n = 512x384$

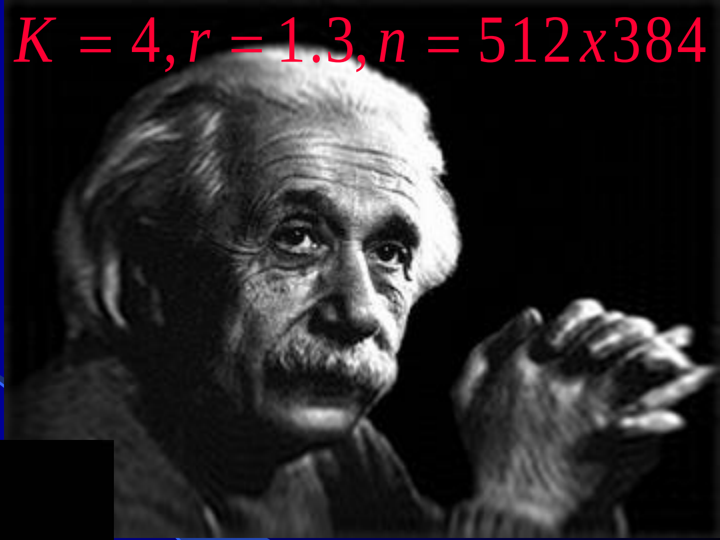

$K = 4, r = 1.3, n = 512x384$

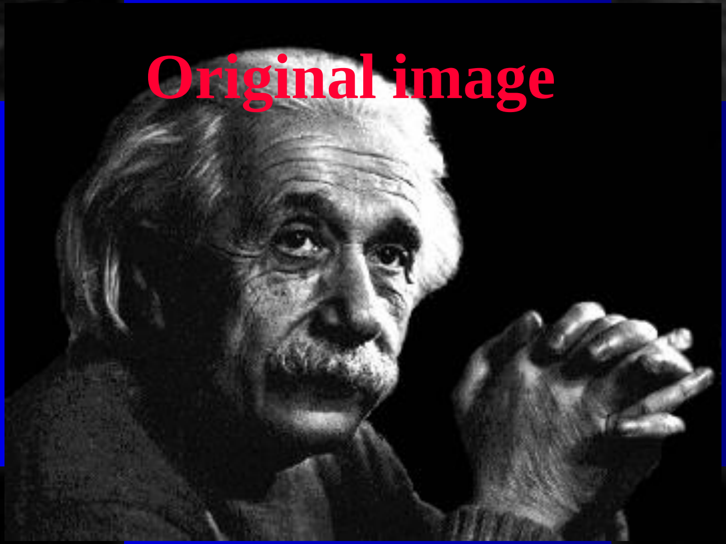

Original image

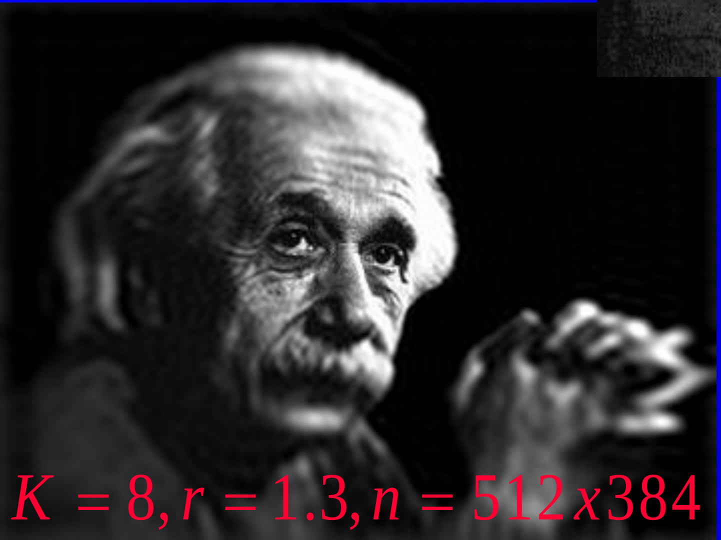

$K = 8, r = 1.3, n = 512x384$

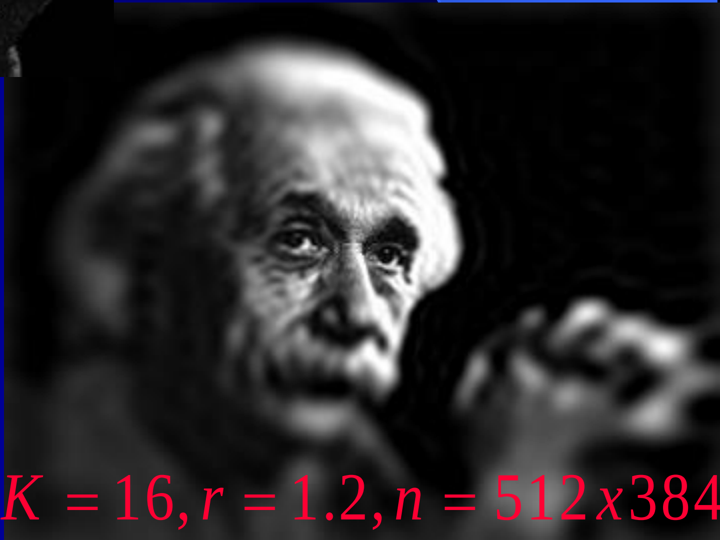

$K = 16, r = 1.2, n = 512x384$

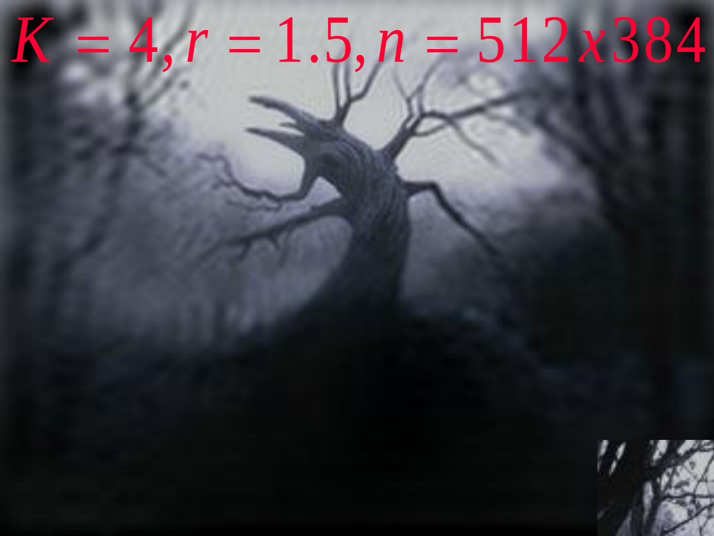

$K = 4, r = 1.5, n = 512x384$


$K = 4, r = 1.3, n = 512x384$


Original image

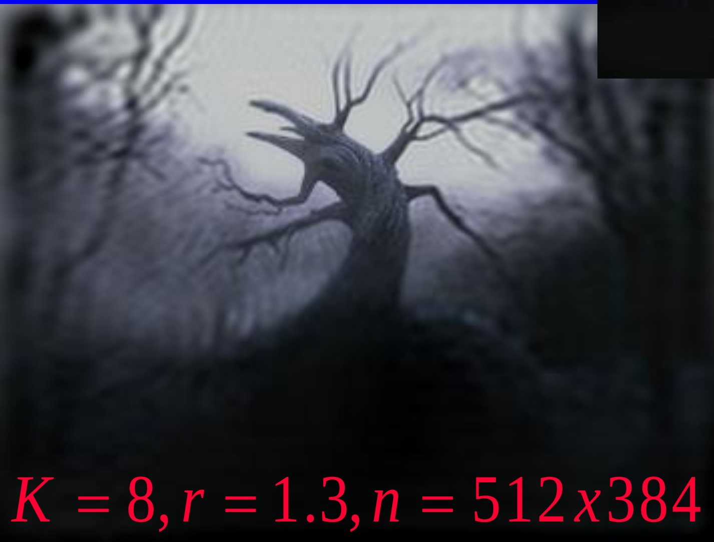

$K = 8, r = 1.3, n = 512x384$

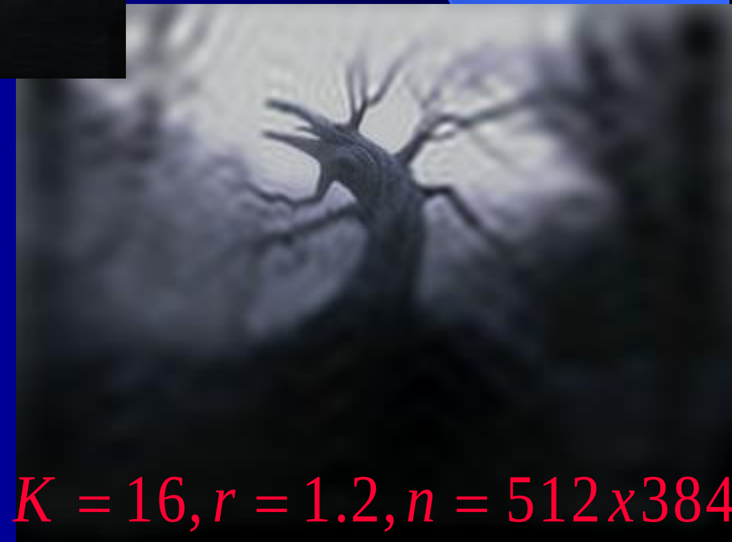

$K = 16, r = 1.2, n = 512x384$


$K = 4, r = 1.5, n = 512x384$


$K = 4, r = 1.3, n = 512x384$


Original image


$K = 8, r = 1.3, n = 512x384$


$K = 16, r = 1.2, n = 512x384$

Conclusion

Hermite foveation allows us to compress useful data, to improve performance of coding/decoding and to use advantages of a time-frequency analysis.